Optimization for power control

(Mostly from the book *Convex Optimization* by Boyd and Vandenberghe)

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Introduction

- When you have to solve an optimization problem, the first step is always to work out to which class of problems it belongs
- different problem types behave very differently
- different methods must be used for different problem types
- never use a black-box solver - it may fail, or at best be very inefficient

Convexity

$$\begin{array}{rl} \text{minimize} & f_0(x) \\ \text{subject to} & f_i(x) \leqslant 0 \\ & a_j^T x = b_j \end{array}$$

- objective is convex
- feasible set is convex
- any local optimum is globally optimal
- not necessarily smooth
- quasiconvex also ok

Linear programming (LP)

$$\begin{aligned} \text{minimize} \quad & c^T x + d \\ \text{subject to} \quad & Gx \preceq h \\ & Ax = b \end{aligned}$$

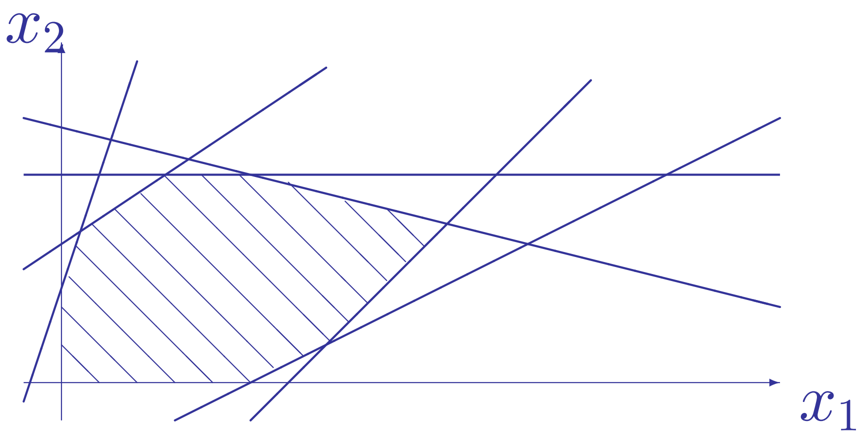
- convex
- standard form

$$\begin{aligned} \text{minimize} \quad & c^T x \\ \text{subject to} \quad & x \succeq 0 \\ & Ax = b \end{aligned}$$

- convert using slack variables
- other problems equivalent - e.g. piecewise linear objective

Linear programming

- ★ has elegant duality theory
- ★ simplex method is fast and accurate
- ★ can solve problems with 1M variables
- ★



Fractional linear programming (FLP)

★

$$
\begin{array}{rl}
\text{minimize} & \frac{c^T x + d}{e^T x + f} \\
\text{subject to} & e^T x + f \succeq 0 \\
& Ax = b
\end{array}
$$

★ if feasible set is nonempty and bounded, can convert to LP

$$
\begin{array}{rl}
\text{minimize} & c^T y + dz \\
\text{subject to} & Gy - hz \preceq 0 \\
& Ay - bz = 0 \\
& e^T x + f = 1 \\
& z \geqslant 0
\end{array}
$$

Quadratic programming (QP)

-

$$\begin{array}{rc} \text{minimize} & x^T P x + 2q^T x + R \\ \text{subject to} & Gx \preceq h \\ & Ax = b \end{array}$$

- P symmetric positive-definite
- is convex
- e.g. least-squares
- e.g. distance between polyhedra

Second-order cone programming (SOCP)

★

$$\begin{array}{rc} \text{minimize} & c^T x \\ \text{subject to} & \|A_i x + b_i\|_2 \leqslant c_i^T x + d_i \\ & Fx = g \end{array}$$

★ is convex

Robust LP

- allow uncertainty in parameters
-

$$\begin{array}{ll} \text{minimize} & c^T x \\ \text{subject to} & a_i^T x = b_i \end{array}$$

- example: $a_i \in \{ \bar{a}_i + P_i u \mid \|u\|_2 \leqslant 1 \}$
- equivalent to

$$\begin{array}{ll} \text{minimize} & c^T x \\ \text{subject to} & a_i^T x + \|P_i^T x\|_2 \leqslant b_i \end{array}$$

- i.e. SOCP

Robust LP - statistical version

- a_i iid Gaussians with known mean and variance
-

$$\begin{array}{ll} \text{minimize} & c^T x \\ \text{subject to} & \Pr\left[a_i^T x \leqslant b_i\right] > \eta \end{array}$$

- equivalent to

$$\begin{array}{ll} \text{minimize} & c^T x \\ \text{subject to} & \overline{a}_i^T x + \Phi^{-1}(\eta) \left\|\Sigma_i^{1/2} x\right\|_2 \leqslant b_i \end{array}$$

- i.e. SOCP
- e.g. portfolio optimization

Generalized fractional linear programming (GFLP)

★

$$
\begin{array}{ll}
\text{minimize} & t \\
\text{subject to} & Cx+d \preceq t(Fx+g) \\
 & fx+g \succeq o \\
 & Ax = b
\end{array}
$$

★ equivalent to

$$
\begin{array}{ll}
\text{minimize} & \max_i \frac{c_i^T x+d_i}{e_i^T x+f_i} \\
\text{subject to} & Gx \preceq h \\
 & Ax = b
\end{array}
$$

★ is quasiconvex

Power control

- the transmitter power control problem is of GFLP type. . .
- where $x_i = P_i$, and other constants are related to SNR, path gain etc.
- is quasiconvex
- is there a statistical version of this?
- is a distributed algorithm possible?