Trip planning in the presence of stochastic delays

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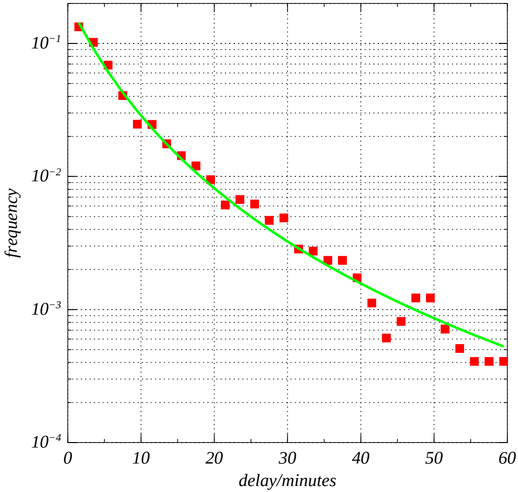
BICS Industrial Complexity Forum 2009-07-10 11:00

BT Research - Adastral Park

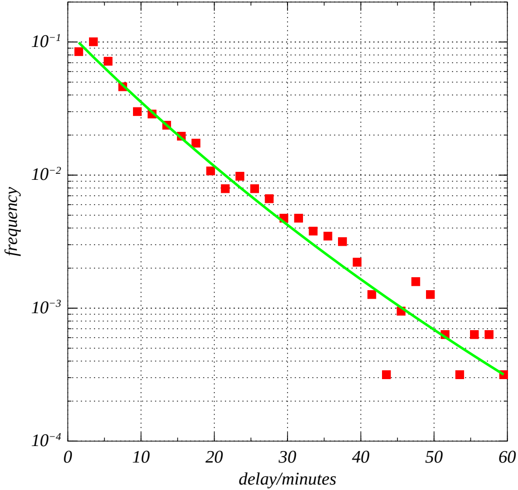
Bath, all departures

$q = 1.288$
$\beta = 0.269$



Bath to Bristol

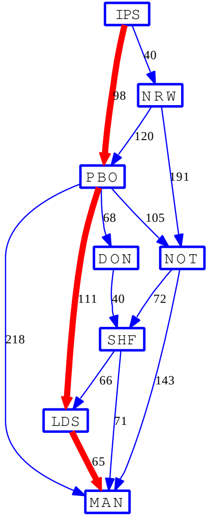
$q = 1.081$
$\beta = 0.128$



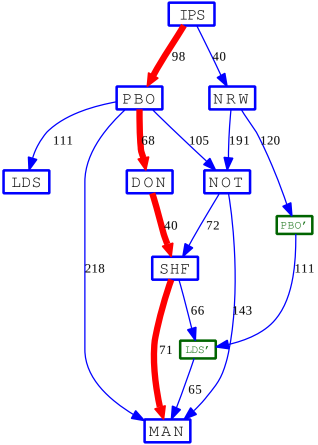
The $q-$exponential law

- Exponential law: $f_\beta(t) \propto \exp(-\beta t)$
- $e_{q,\beta}(x) := (1/Z)(1+\beta(q-1)x)^{1/(1-q)}$, $\beta > 0, 1 < q < 2$
- $Z := \frac{1}{\beta(2-q)}$
- mean $\mu := \frac{1}{\beta(3-2q)}$
- $\lim_{q\to 1} e_{q,\beta}(t) = \exp(-\beta t)/Z$
- large q gives a power-law (long tail)

Short paths in a weighted digraph

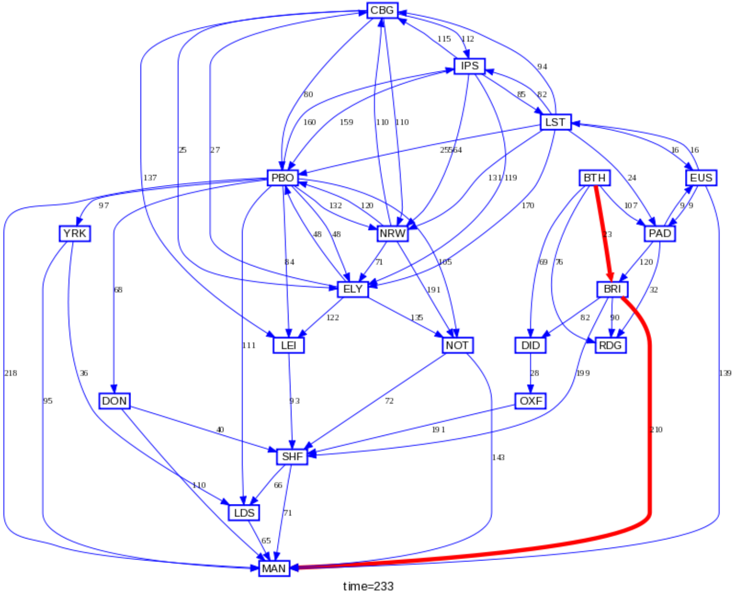


time=275

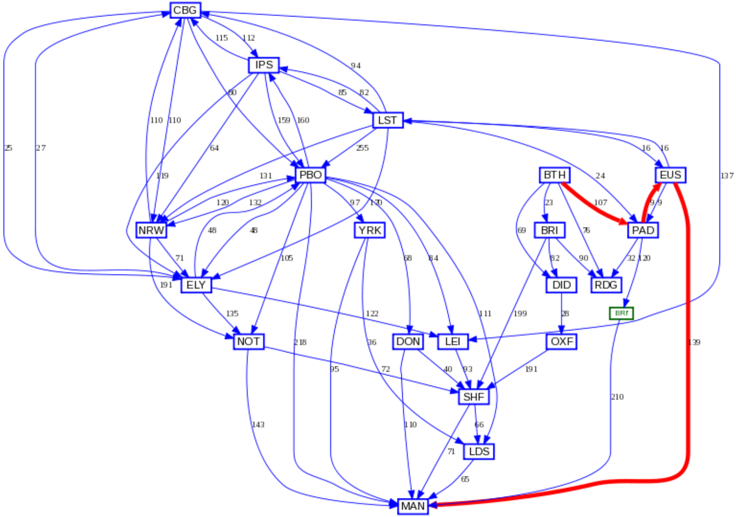


time=278

Bath to Manchester, shortest mean time

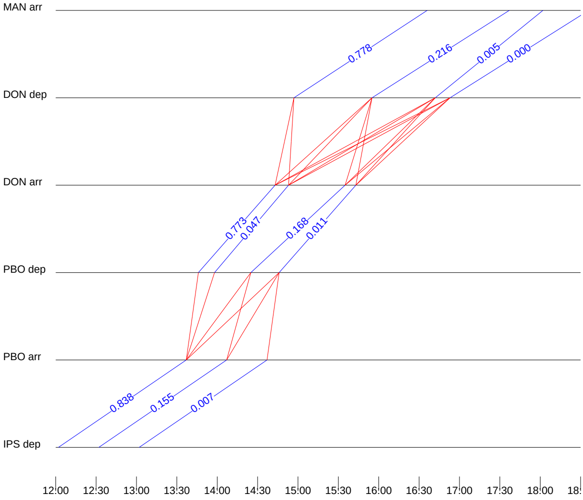


Bath to Manchester, second shortest mean time



time=256

IPS→PBO→DON→MAN dep. 1200



IPS→PBO→DON→MAN dep. 12:00

```
IPS 12:02 -> PBO 13:37 0.8380
   PBO 13:46 -> DON 14:43 0.7731
      DON 14:57 -> MAN 16:36 p=0.7377
      DON 15:55 -> MAN 17:37 p=0.0353
   PBO 13:58 -> DON 14:53 0.0466
      DON 14:57 -> MAN 16:36 p=0.0405
      DON 15:55 -> MAN 17:37 p=0.0061
   PBO 14:25 -> DON 15:35 0.0173
      DON 15:55 -> MAN 17:37 p=0.0169
      DON 16:42 -> MAN 18:02 p=0.0004
   PBO 14:46 -> DON 15:43 0.0009
      DON 15:55 -> MAN 17:37 p=0.0009
IPS 12:32 -> PBO 14:07 0.1551
   PBO 14:25 -> DON 15:35 0.1505
      DON 15:55 -> MAN 17:37 p=0.1468
      DON 16:42 -> MAN 18:02 p=0.0036
   PBO 14:46 -> DON 15:43 0.0041
      DON 15:55 -> MAN 17:37 p=0.0039
      DON 16:42 -> MAN 18:02 p=0.0002
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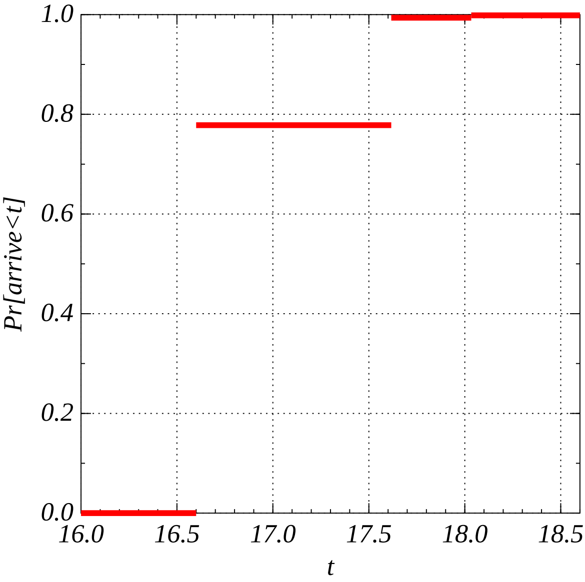
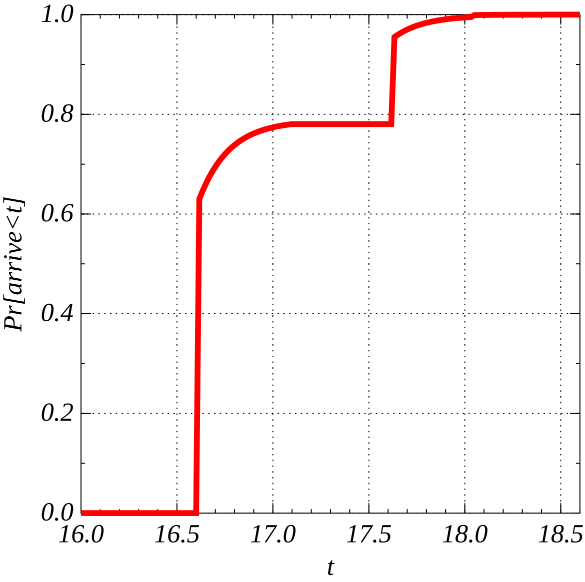
IPS→PBO→DON→MAN dep. 12:00



IPS→PBO→DON→MAN dep. 1200



Problem formulation

- Given: a weighted digraph g, a timetable $\tau(n_0, n_1)$ for each arc $(n_0, n_1) \in g$, an arrival time α, and parameters $\tau > 0, \epsilon > 0$.
- To find: a route ρ and maximal departure time t such that Prob[arrival after $\alpha + \tau$] $< \epsilon$

Algorithm

- Phase 0: find set P of the 3 or 4 paths of shortest mean time
- Phase 1: for each path $p \in P$, and for a given start time, propagate all probabilities through the graph
- Compute probability $\rho = \text{Prob}[\text{arrival after } \alpha + \tau]$
- If $\rho > \epsilon$, repeat with an earlier start time.

IPS→MAN arr. 19:00

```
Iter 1:  Probability of arriving by 19:00 is 99.9%
Ipswich             12:02 -> Peterborough           13:37
Peterborough        14:56 -> Doncaster              15:55
Doncaster           16:42 -> Manchester Piccadilly  18:02

Iter 2:  Probability of arriving by 19:00 is 98.3%
Ipswich             12:02 -> Peterborough           13:37
Peterborough        14:56 -> Doncaster              15:55
Doncaster           16:53 -> Sheffield              17:20
Sheffield           17:40 -> Manchester Piccadilly  18:36

Iter 3:  Probability of arriving by 19:00 is 95.7%
Ipswich             12:02 -> Peterborough           13:37
Peterborough        14:56 -> Doncaster              15:55
Doncaster           17:01 -> Leeds                  17:36
Leeds               17:55 -> Manchester Piccadilly  18:49
```

Reference

K M Briggs & C Beck, *Modelling train delays with q-exponential functions* Physica **A 378**, 498–504 (2007).